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The η-invariant of the cone on a nonsingular hypersurface in a complex projective space

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Atiyah, Patodi and Singer[1] defined an invariant η for oriented (4n-1)-dimensional Riemannian manifolds. The purpose of this note is to announce the determination of η of some particular manifolds. Namely, let $V^{2n}(d)$ be the cone on a nonsingular hypersurface of degree d in complex projective 2n-space CP^{2n}. Let $M^{4n-1}(d) = V^{2n}(d) \cap S^{4n+1}$, where S^{4n+1} is the unit sphere in C^{2n+1}. Then $M^{4n-1}(d)$ is an oriented (4n-1)-dimensional Riemannian manifold (the metric is the one induced from that of C^{2n+1}). The result is

Theorem

$$\eta(M^{4n-1}(d)) = d(d-1)^{2n} \frac{2^{2n+1}}{(2n+1)!} a_{2n+1} - \operatorname{sgn} F$$

where F is the manifold defined by the equation; $z_0^d + \cdots + z_{2n}^d = 1$ in C^{2n+1} and a_{2n+1} is the (Euler) number defined by

$$\frac{2}{e^t+1} = \sum_{n=0}^{\infty} a_n \frac{t^n}{n!} .$$

Example

Let $P^3 = S^3/\pm 1$ be the projective space with the standard metrc. Then it is easy to see that $\eta(P^3)=0$. On the other hand, let $K^3 = V_2 \cap S^5$, where V_2 is the locus of $z_0^2 + z_1^2 + z_2^2 = 0$ in C^3. Then it is known that K^3 is diffeomorphic to P^3, but we have $\eta(K^3)=\frac{1}{3}$ by the Theorem.

Reference

[1] M.F.Atiyah, V.K.Patodi and I.M.Singer, Spectral asymmetry and Riemannian geometry, Math. Proc. Camb. Phil. Soc. (1975), 77, 1-6.

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